CASSIOPEIA'S TOE

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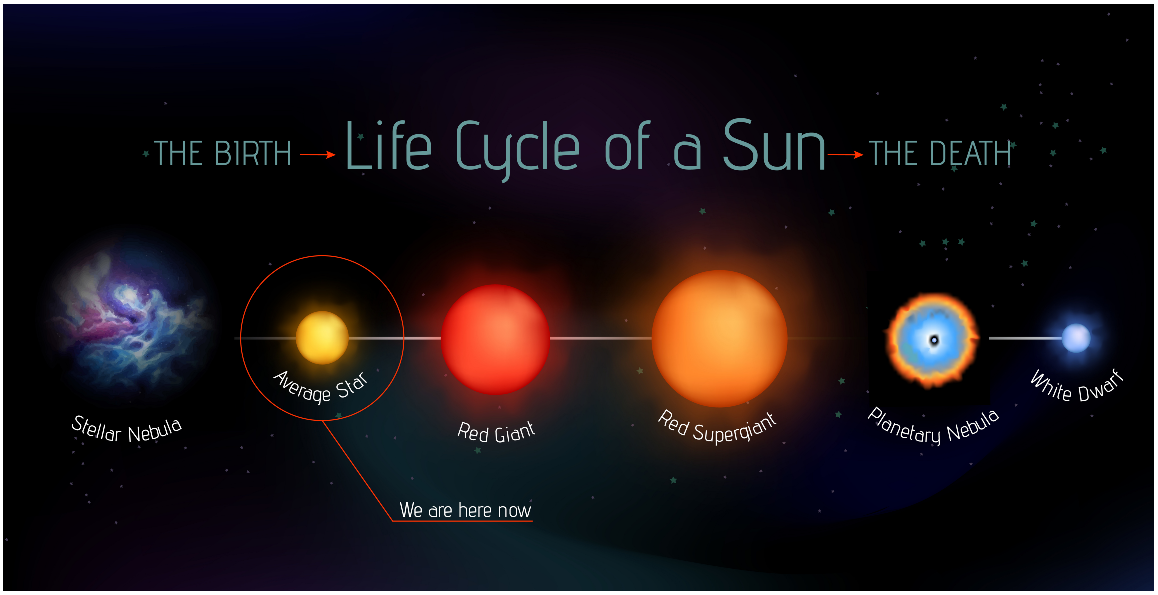
White Dwarf Stars

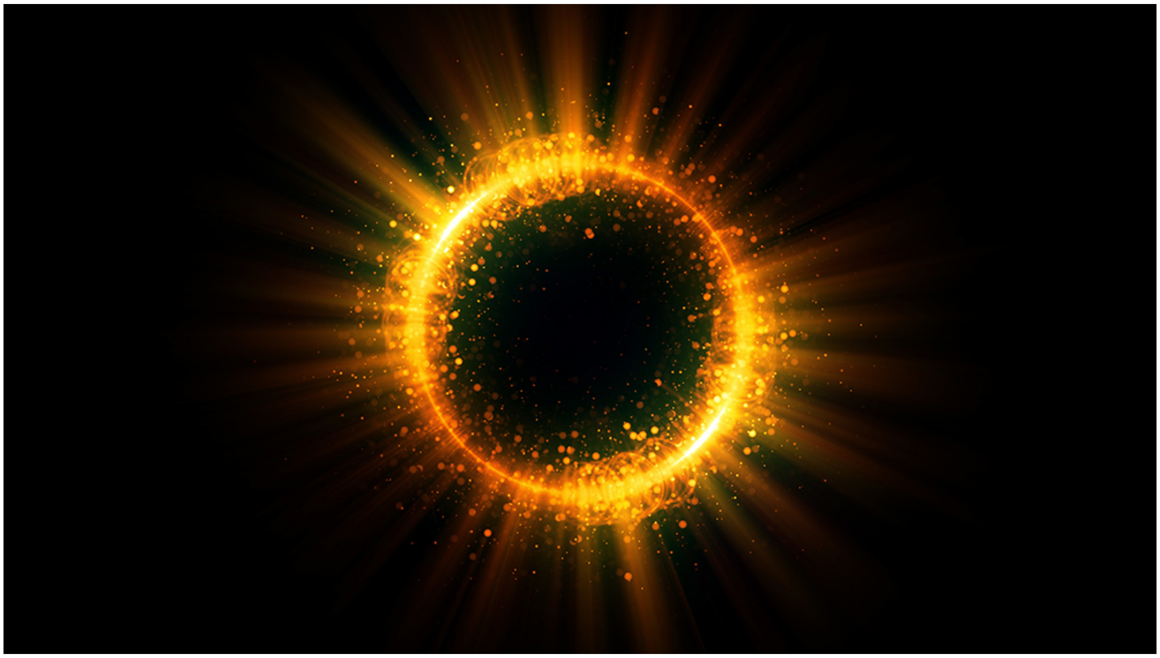


When stars with a mass of ten times that of our sun or less reach the end of their life, they become white dwarf stars that have a remnant mass of our sun within the size of the earth. After the hydrogen-fusing period of a main-sequence star of this size ends, such a star will expand to become a red-giant during which it fuses helium to form carbon and oxygen in its core. If a red giant has insufficient mass to generate the core temperatures required to fuse carbon and oxygen (around 1 billion K), then an inert mass of carbon and oxygen will build up at its center. After such a star sheds its outer layers, it will leave behind a core, which is the remnant white dwarf.

White dwarfs resist gravitational collapse primarily through electron degeneracy pressure as opposed to thermal pressure that works for younger, larger, hotter stars. Electron degeneracy pressure is the result of the Pauli Exclusion Principle -- electrons are fermions and obey Fermi-Dirac statistics. So no two electrons can exist in the same state. So although gravity squeezes the atoms in the star, the electrons cannot go to lower energy states that are already occupied by other electrons. The ***Chandrasekhar limit*** is the mass above which electron degeneracy pressure in the star's core is insufficient to balance the star's own gravitational self-attraction. Consequently, a white dwarf with a mass greater than the limit is subject to further gravitational collapse, evolving into a different type of star, such as a neutron star. Those with masses under the limit remain stable as white dwarfs.

When the mass is higher than the limit, the atomic electrons are captured by the nucleus where they combine with protons to form neutrons and emit neutrinos to balance the lepton number requirement.

Neutron Stars



Neutron stars are the smallest and densest stars, and they have a radius on the order of 10 kilometres (6.2 mi) and a mass between 1.4 and 2.16 times the mass of our sun. They result from a supernova explosion of stars with a mass between 10 and 30 times the mass of our sun. The remnant material undergoes gravitational collapse that compresses the core past white dwarf density all the way down to the density of atomic nuclei. (As above, the atomic electrons combine with protons and emit neutrinos to become neutrons.) If the mass of the original star is greater than about 30 times the mass of our sun, then the collapse proceeds past a neutron star and into a black hole.

The **Tolman–Oppenheimer–Volkoff limit** (or **TOV limit**) is an upper bound to the mass of cold, nonrotating neutron stars, analogous to the ***Chandrasekhar limit*** for white dwarf stars. Observations suggest that the limit is close to 2.17 solar masses. At masses greater than this, the object becomes a black hole.

The small radius and high mass suggests densities beyond nuclear densities although we don't know what form the nuclear particles take to achieve these densities.

Now let's address the relativistic problem of how any black hole can form.

Black Holes

Most people can visualize a Black Hole – it is a dark region sitting in space obstructing stars. And the usual explanation for the blackness is that stuff falls in but gravity is so strong that not even light can escape – so nothing ever comes out.

And to quote Luke in ***The Last Jedi*** … “every word in that sentence is wrong!”

You might – shoulda – oughtta know that time slows down in a gravity well… If I have a clock on the ground floor of a tall building and one on the 10th floor, the one on the ground floor runs slower.

So let’s sit in a space ship comfortably away from a Black Hole (non-rotating) and watch a clock fall in under the pull of gravity. As it falls, we are not surprised that it runs slower than clocks on the ship with us because we know that the stronger gravity is, the slower that clock will run. But we are a little surprised to see that it only accelerates downward until it gets to about 4 times the radius of the black hole. Then it no longer appears to be accelerating downward – in fact, it appears to be slowing down. And it runs slower and moves slower and turns increasingly redder and dimmer as it falls… until it STOPS entirely – it stops running AND it stops falling entirely. If we could just barely still see it, it would be stationary in space at a point a certain distance away from the center of the Black Hole and that distance is called the event horizon or the Schwarzschild Radius. It is the point at which time stops as viewed by an external observer. It is the point at which the "escape" velocity is equal to the speed of light. It is the point at which it would take infinite "proper" acceleration for the object to move away from the black hole. And it took the clock forever to get that far down.

So if nothing can ever fall into a black hole from an external perspective, why do we talk about Black Holes "eating" stars and the surrounding gas? In fact, how do black holes ever form in the first place if time stops at the event horizon (relative to external observers). The usual answer is that time doesn't stop for the matter falling in, so it just keeps going. That may be true for the falling matter, but for us on the outside, it will take infinite time for it to get to the event horizon, so, in our timeline, it NEVER gets inside the event horizon – at least not by crossing the existing event horizon.

Stephen Hawking (RIP) and Leonard Susskind have suggested (after decades of arguing about it) that matter falling towards a black hole, eventually just gets smeared out on the surface of the Event Horizon. But they still talk about what's inside that horizon… perhaps a singularity at the center, for example.

It is also common to talk about the gravity generated by all the matter in the black hole… and we unquestionably see the effects of the force of gravity on the stars and gasses around them. But if gravitons (or something like them) are the propagator of gravity and if they travel at the speed of light, how do they ever make it out to influence anything?

The answer that they are virtual gravitons and can violate the limitations on the speed of light – as well as conservation of energy – and other physical laws is pretty unsatisfying. And we will have a close look at this question in a little while.

For now, we propose a method for building a black hole that solves the problem of time stopping at the event horizon for all external observers – and that is us – external observers.

Here is a recipe for building a black hole from the viewpoint of an external observer…

Although it may be possible to have matter density greater than the density of nucleons in an atomic nucleus, it isn't necessary for this recipe. So we will just skip the micro-black-hole possibility since it requires some exotic form of matter that we are only theorizing about.

It is likely that matter in neutron stars that have masses greater that 2-3 times the mass of our sun is in some exotic state, but for now let's address neutrons that are packed at standard nuclear densities. Even at nuclear densities, if a neutron star has a mass greater than about 1.8 x 10^{31} kilograms (10 times the mass of our sun), it will have an event horizon – that is to say, it is a black holes. And for this recipe, let's assume that this is the first event horizon for this object. This type of black hole… a stellar black hole… has a density similar to that in the nucleus of a typical atom… about 2.3 x 10^{17} kg/m^3 … and a Schwarzschild radius of about 26.5 km.

Additional in-falling mass will accumulate on the event horizon as its time (as viewed by us) slows and stops. But with each new accumulation of mass on that event horizon, the overall increase in mass creates a new event horizon farther out. And additional in-falling mass will accumulate on that event horizon. This will create still another event horizon farther out. Notice that each new event horizon doesn't change the fact that time is still stopped on all smaller event horizons. So the Black Hole grows like an onion with layers of event horizons in its interior. So from the point of view of ANY external observer, none of the mass ever crosses any event horizon, and there is never the need for a singularity at the center with infinite mass density.

For example, let's revisit that description of a neutron star that is barely a black hole. Then If we accumulate more mass – still at nuclear density -- so that our physical volume grows from 26.5 km to 26.6 km – then the new Event Horizon will be at 26.8 km. It turns out that the Event Horizon (Schwarzschild radius) will always grow faster than then accumulated mass that fits inside that volume at just standard nuclear density, and therefore all accumulated mass will be inside the new event horizons.

Again, it is probably true that the matter inside the event horizons doesn't stay in the form of neutrons obeying the exclusion principle, but we have shown that exotic forms of matter are not necessary to form a Black Hole. And to an external observer, it really doesn't matter what form the internal matter takes – from quark-gluon plasma to something even more exotic. Remember though that all the matter particles that we know are fermions, and there can be no singularity unless the exclusion principle is violated or the fermions somehow morph into bosons. Later we will explore how these possibilities might look in the wormhole view.

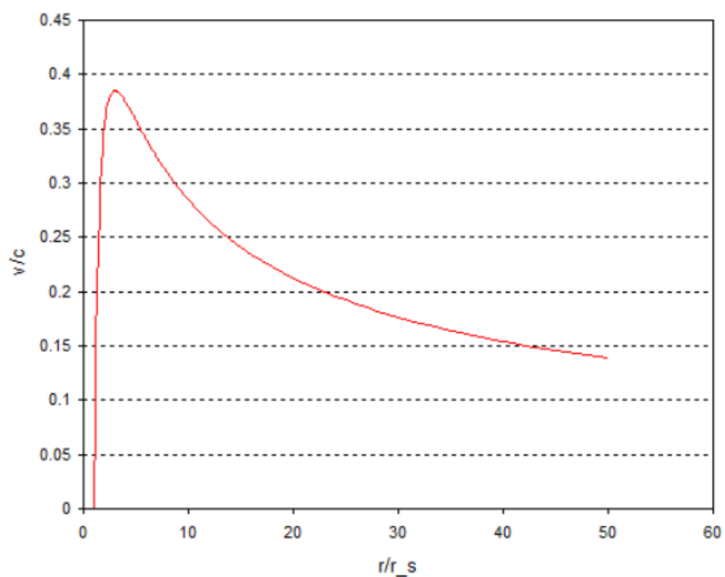
QUANTITATIVELY

If you would like a better picture of WHEN the in-falling object stops accelerating inward (apparently) and starts to slow down (apparently to a distant observer)

If you sit a long way from the black hole and watch an object falling into it from far away then the velocity of the object will be related to distance from the black hole by:

$$v = \left(1 - \frac{r_s}{r}\right)\sqrt{\frac{r_s}{r}}c \qquad (1)$$

where r_s is the Schwarzschild radius. If we graph the velocity as a function of distance from the black hole we get:



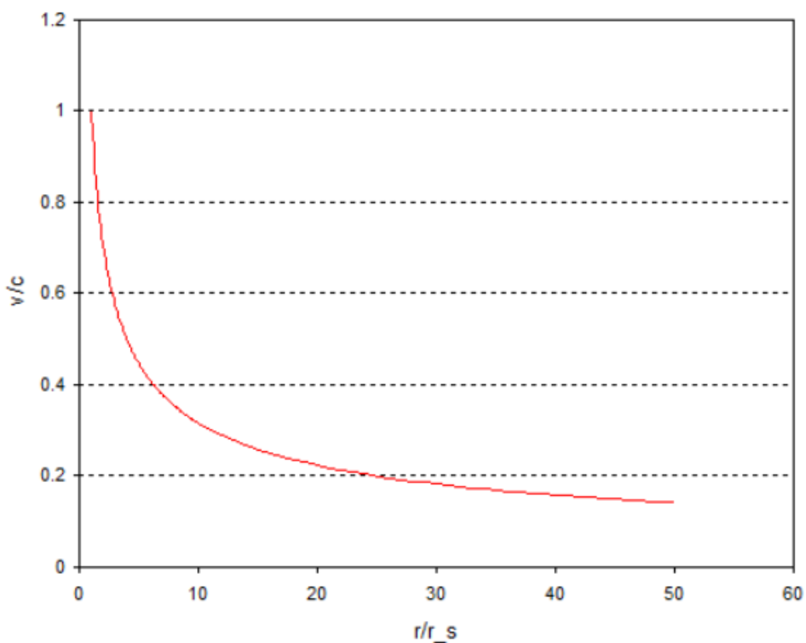
The x axis shows distance in Schwarzschild radii while the y axis is the speed as a fraction of the speed of light. The speed peaks at about $0.38c$ then falls as you get nearer to the event horizon and falls to zero at the horizon. This is the source of the notorious claim that nothing can fall into a black hole.

For a different perspective, let's ask what the velocity looks like to a large number of "proper" observers stationed at closer and closer positions to the black hole, and with each observer watching the object go by. For these observers of course there is no time dilation at their location…

An alternative strategy might be to hover at some distance r from the black hole and measure the speed at which the falling object passes you. These observers are known as shell observers. If you do this you find a completely different variation of speed with distance:

$$v = \sqrt{\frac{r_s}{r}}c \qquad (2)$$

This time the variation of the speed with distance looks like:



and this time the speed goes to c as you approach the horizon. The difference between the two is because time slows down near a black hole, so if you're hovering near the event horizon velocities look faster because your time is running slower. You might be interested to note that the velocity calculated using equation (2) is equal to the Newtonian escape velocity. The event horizon is the distance where the escape velocity rises to the speed of light.

The last observer is the falling observer i.e. the one who's falling into the black hole. But here we find something even stranger. The falling observer will never observe themselves crossing an event horizon. If you're falling into a black hole you will find an apparent horizon retreats before you as you fall in and you'll never cross it. You and the horizon will meet only as you hit the singularity.

And for light as opposed to a massive object, it's velocity looks like this…

To show this let's take an example. If you solve Einstein's equations for a spherically symmetric mass you get the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2\, dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

In this equation r is the distance to the black hole (the radius) and t is time (what you measure on your wristwatch). θ and ϕ are basically longitude and latitude measurements. The quantity ds is called the *interval*. r_s is the radius of the event horizon. The co-ordinate system strictly speaking is the one used by an observer at infinity, but it's a good approximation as long as you are well outside the event horizon.

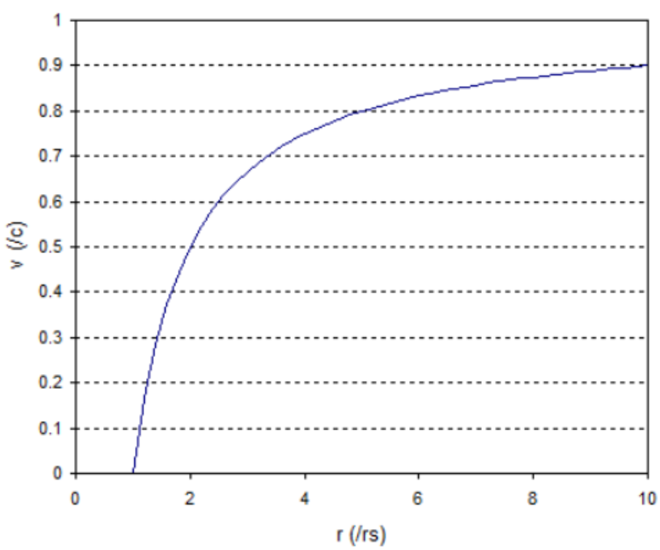
For light rays ds is always zero, and we can use this to calculate the velocity of the light ray. For simplicity let's take a ray headed directly towards the black hole, so the longitude and latitude are constant i.e. $d\theta$ and $d\phi$ are both zero. This simplifies the above equation to:

$$0 = -\left(1 - \frac{r_s}{r}\right) c^2\, dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)}$$

The velocity of the light, v, is just the rate of change of radius with time, dr/dt, and we get this by a quick rearrangement:

$$\frac{dr}{dt} = v = c\left(1 - \frac{r_s}{r}\right)$$

The variation of the velocity of light with distance from the black hole looks like:



At large distances (large r) the velocity tends to 1 (i.e. c) but close to the black hole it decreases, and falls to zero at the event horizon.

So, to calculate the speed of light in your co-ordinate system solve the Einstein equations to get the metric, set ds to zero and solve the resulting equation - sounds easy, but it rarely is!

The graphs and formulas are extracted from the Physics Stack Exchange, and should be credited to…

https://physics.stackexchange.com/users/1325/john-rennie

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John Rennie

My career in science started with a degree in Natural Science at the University of Cambridge specialising in quantum chemistry. Then I did a PhD, also at Cambridge, in solid state photochemistry. To my surprise my PhD is listed on the web even though the World Wide Web didn't exist at the time!

After finishing my PhD I worked as a colloid scientist for Unilever Research. Now I'm getting old and grey I work part time as a computer nerd, and answer questions on the Physics Stack Exchange to stop my brain from atrophying.

I'm mostly interested in general relativity though I'll have a go at anything related to quantum mechanics as well. Next on my bucket list is to learn quantum field theory though that is turning out to be a challenge! I hang around in the chat room and I'm happy to answer questions there as well. Just ping me if you want my attention.