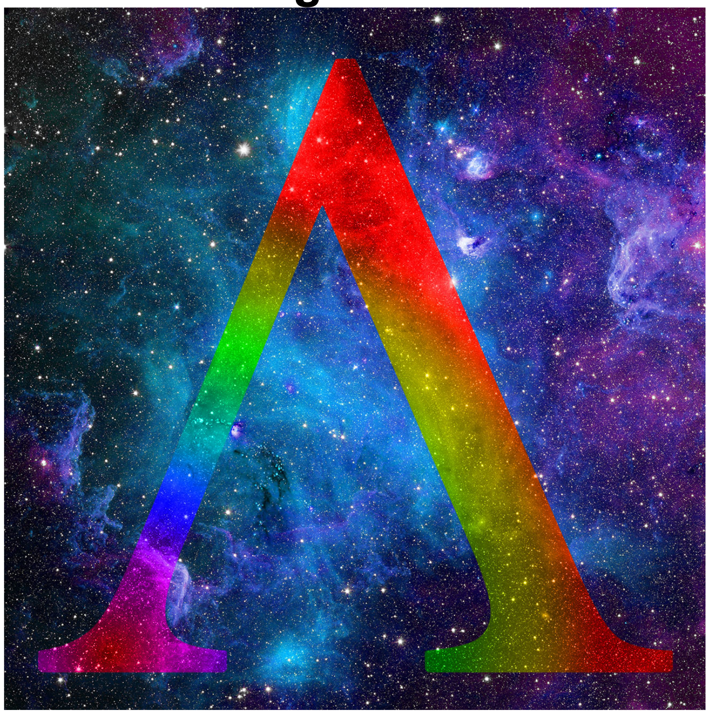
CASSIOPEIA'S TOE

18

Cosmological Constant



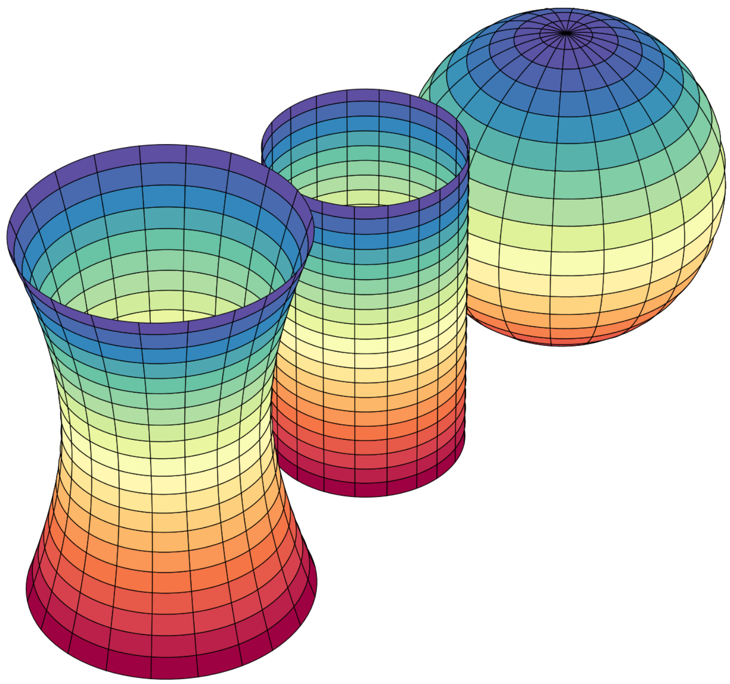
Type 1a supernova is a type of supernova that can occur in a binary star system in which one of the stars is a white dwarf. The other star can be anything from a giant star to an even smaller white dwarf. White dwarfs are star remnants that do not have sufficient mass to drive nuclear fusion of its remaining elements. But as the white dwarf in the system steals mass from its companion, its dead core gradually reaches ignition temperature for carbon fusion to reignite. And when it does, a runaway reaction occurs causing the small star to explode in a supernova.

Because of the uniform mass of these type 1a supernovas, this explosion produces perfectly consistent peak luminosity. This makes these supernovas a "standard" candle" – an object whose absolute brightness we know regardless of when or how far away it occurs.

In 1998, two independent teams used this constancy of luminosity to measure the distances to various type 1a supernovas. Then they compared this distance to the cosmological redshift of these objects, which measures how much the universe has expanded since the supernova occurred. Cosmologists at the time expected that the recession velocity of the more distant objects would be decreasing due to the gravitational attraction of the matter in the universe. However, they found the opposite. They found that the expansion of the universe is increasing with time.

In General Relativity, an accelerated expansion is accounted for by a positive value of the cosmological constant Λ (lambda). This is equivalent to the existence of a positive vacuum energy (also called dark energy).

Consider this figure…



In the cylinder, notice that the shape of the geodesics as we move vertically from the waist of the object remain parallel. This cylinder has zero curvature. In General Relativity this space would be devoid of all matter and also devoid of any vacuum energy. The analog to this cylinder in 4-dimensional spacetime is called Minkowski space. (Hermann Minkowski was one of Einstein's professors. He showed that Einstein's Special Theory of Relativity could be understood geometrically as a theory of 4-dimensional spacetime called Minkowski Space)

In the sphere, as we move vertically from the waist of the object, the geodesics converge towards the pole. This sphere has positive curvature. In the absence of matter and energy, this space with positive curvature represents a space with a positive vacuum energy and a positive cosmological constant. Its analog in 4-dimensional spacetime is called a de Sitter space (after Willem de Sitter – a close associate of Einstein and Minkowski).

In the hyperboloid on the left, the geodesics as we move vertically from the waist of the object diverge. This hyperboloid has negative curvature. In the absence of matter and energy, this space with negative curvature represents a space with a negative vacuum energy and a negative cosmological constant. Its analog in 4-dimensional spacetime is called an ANTI-de Sitter space.

These 4-dimensional analogs to the surfaces shown above are the exact solutions to Einstein's field equations for an EMPTY (void of matter) universe with a negative, zero, or positive cosmological constant. And these in turn correspond to ta universe with negative, zero, or positive vacuum energy.

An intrinsic curvature of spacetime in the absence of matter or energy is modeled by the cosmological constant in general relativity. This corresponds to the vacuum having an energy density and pressure. This spacetime geometry results in initially parallel timelike geodesics diverging, with spacelike sections having positive curvature. (Not to be inferred from the 3-dimensional figures above)

The attractive force of gravity is due to the fact that matter and energy tend to give space negative curvature. This induced curvature due to the presence of matter and energy is superimposed on the intrinsic curvature of space caused by the existence of vacuum energy or equivalently a cosmological constant.

So the effect of the presence of matter and energy counteract the presence of a positive vacuum energy (which causes negative pressure). And the resultant overall spacetime curvature is what we witness.

Currently, we have measured a small and positive overall cosmological constant.

SOME TECHNICAL NOTES FROM WIKIPEDIA
The cosmological constant Λ appears in Einstein's Field Equation in the form

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The Ricci tensor/scalar *R* and the metric tensor *g* describe the structure of spacetime,

The stress-energy tensor *T* describes the energy and momentum density and flux of the matter in that point in spacetime,

And the universal constants *G* and *c* are conversion factors that arise from using traditional units of measurement.

When *Λ* is zero, this reduces to the field equation of general relativity without a cosmological constant of course.

When *T* is zero, the field equation describes empty space (the vacuum).

The cosmological constant has the same effect as an intrinsic energy density of the vacuum, ρ_{vac} (and an associated pressure).
The current value for Lambda is…

$$\Lambda = 1.1056 \times 10^{-52}\,\mathrm{m}^{-2}$$

A positive vacuum energy density resulting from a cosmological constant implies a negative pressure, and vice versa. If the energy density is positive, the associated negative pressure will drive an accelerated expansion of the universe, as observed.

Using the upper limit of the cosmological constant, the vacuum energy of free space has been estimated to be 10^{-9} joules per cubic meter.

1 joule = 6.242 X 10^{12} MeV

So the vacuum energy of space is observed to be about 6242 MeV per cubic meter. And the calculated value (from quantum field theory) is 10^{113} joules per cubic meter.

That is a big difference – known as the cosmological constant problem.

In field theory, fields are modeled as having a simple (quantum) harmonic oscillator at every single tiny point of space. The lower limit is of some debate, but the smallest point of space is often taken to be the Planck Volume. Then, the vacuum energy is calculated by summing over all known quantum-mechanical fields, taking into account interactions and self-interactions between the ground states, and then removing all interactions below a minimum "cutoff" wavelength to reflect that existing theories break down and may fail to be applicable around the cutoff scale. This method leads to the enormous value shown above (10^{113} joules per cubic meter).

IN THE WORMHOLE VIEW, this discrepancy can possibly be explained. The calculations from QED assume that space is continuous at least down to the Planck length, but in the wormhole view, we assume space is connected by wormholes and this can effectively modify the “volume” of each cubic meter of space. Thus when we calculate the emergence of vacuum energy in the form of virtual particles being created and annihilated because of the Uncertainty Principle, we must reduce the volume of space by a factor determined by the “average length” of free space wormholes.

If the general size of space molded, affected by, and occupied by an electron corresponds to its de Broglie wavelength, which is on the order of one nanometer, then we can simply compare the number of Planck volumes in a cubic meter to the number of wormhole-shaped volumes in the same space.

There are about 10^{106} Planck Volumes in a cubic meter
There are about 10^{27} Electron-Sized Wormhole volumes in a cubic meter

This immediately reduces the value of the calculated cosmological constant by about 10^{80}.

Next let's consider the Uncertainty Principle, and let's assume that our 10^{27} quantum harmonic oscillators in each cubic meter of space are behaving according to the restrictions of the Uncertainty Principle…

$$\Delta E \Delta t \sim \hbar$$

An electron has rest energy of 0.511 MeV, So it can come into existence for no more than about 10^{-22} seconds before it must disappear again. But how long is it on average before that same point spawns another electron?

If we work backwards from the observed cosmological constant, there can only be about 12,000 electrons per cubic meter at any one time. And since we have 10^{27} possible points of production at any one time, each point can only spontaneously produce an electron every 10^{23} cycles – or roughly once per second for each Electron-Sized Wormhole volume. This yields a cosmological constant equivalent to that observed.

Not surprisingly, this procedure has yielded a "dark energy" density of the universe in good agreement with observation when compared to the 1000 MeV per cubic meter for matter.

SIDE NOTES:

As space expands, the density of matter drops as R^3
and the energy density of photons (radiation) drops as R^4
(R^3 because of the volume growth and another factor of R for the cosmological redshift.)
Meanwhile the intrinsic energy density of the vacuum remains constant.

So it is easy to see that while the early universe was dominated by radiation density, after a while it became dominated by matter, and recently (3-4 billion years ago) it became dominated by the vacuum energy.